Advantage and Implementation Considerations of Shaped OFDM Signals

Dragan Vuletic , San Diego State University & Signum Concepts
Wade Lowdermilk, BAE Systems
fred harris, San Diego State University

ABSTRACT: This paper presents the structure and measured performance of a MODEM designed to implement a variant of OFDM known as shaped OFDM. Individual complex sub-carriers matching the mutually orthogonal tones of a Fourier transform with time span of MT seconds form a standard OFDM signal. The amplitudes of the orthogonal sinusoids are obtained by a DFT of uniformly spaced samples of the time function. An interfering tone within the frequency span of the OFDM signal may interfere with all the OFDM channels. The interference can be isolated to a few OFDM channels by replacing the rectangle envelope with a shaped envelope. The shaping controls the side lobe levels of the individual channel spectra and thus suppress the projection of an inter channel interfering signal. To maintain a fixed system throughput, the time span of the OFDM frame is lengthened and the adjacent frames are overlapped.

INTRODUCTION: OFDM modems offer efficient use of channel bandwidth by permitting spectral overlap of the individual QAM modulated sub carriers that form the composite signal. The overlapped carriers correspond to the mutually orthogonal basis set of a DFT with modulation and demodulation performed by an IFFT and FFT respectively. The overlapped spectra are translated spectral sinc functions with spectral peaks located on each other's zero crossings. There are channels that contain one or more narrow band interfering sinusoids within the frequency span of the OFDM bandwidth. If an interfering sinusoid has an integer number of cycles per frame interval it corresponds to the frequency of one of the basis set carriers hence exhibits zero projection on the remaining basis vectors. On the other hand, when the interfering sinusoid has a non-integer number of cycles per frame interval it contributes a component to every OFDM carrier. The projections into the OFDM channels are due to the side-lobes of the sin(x)/x located at the non-bin centered interfering signal. The spectral side lobes can be reduced to arbitrarily small levels by use of smooth envelopes, such as a standard window or the impulse response of a low pass prototype filter, applied to the basis signal set. There are two spectral effects of shaping the DFT basis vectors. The first is the desired reduction in side lobe levels, and the second is a widening of the spectral main lobe.

Specifically, for a time series of duration MT, with say M=64, when we suppress the side lobes to approximately -80dB the spectral main lobe width is increased by a factor of 4 from $f_S/(M)$ or 1/64 to 4 f_S/M or 1/16. We return the spectral width of the shaped time signal to the width of the rectangle window time signal by increasing the signal time duration from MT seconds or 64 samples to 4 MT seconds or 256 samples. The envelope and spectrum of two shaped OFDM signals is illustrated in figure 1.

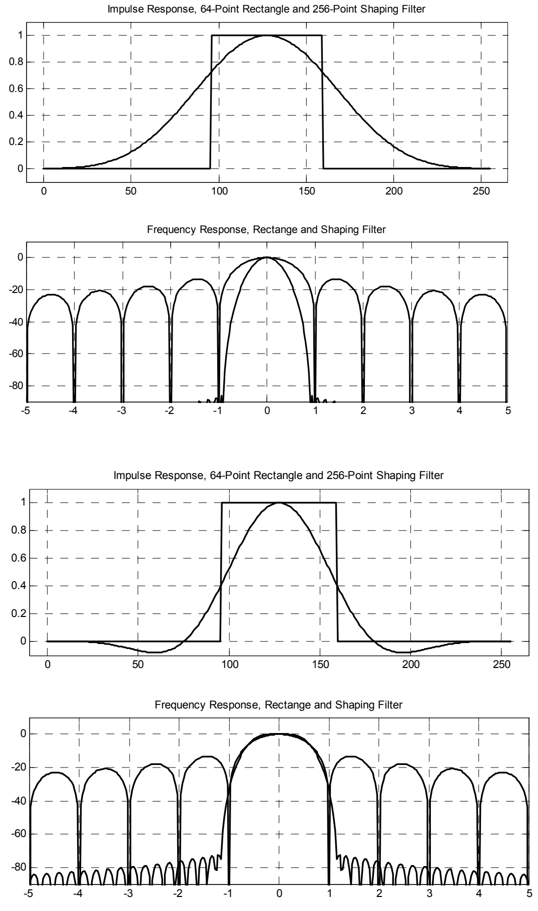

Figure 1. Time and Frequency Responses of Window and of Low-Pass Filter Shaping Functions.

We have employed envelope shaping to control spectral side-lobe levels and the 4-times increased time support to return the main lobe width to its original width of 4 fs/(4M) Hz. The 4-times longer time support permits a 4-times longer FFT with a resolution of fs/256 rather than the original fs/64. As we have preserved the original spectral bandwidth we also elect to preserve the original spectral resolution, which defines the channel center frequencies. We accomplish this by sub-sampling the available spectrum, keeping every 4-th higher resolution frequency bin, specifically bins 4k. The spectral 4-to-1 sub-sampling causes 4-fold time domain aliasing of the shaping filter impulse response. The 4-to-1 time aliasing or time folding is equivalent to a polyphase partition of the shaping filter and of the corresponding matched filter. The polyphase filter folds the 4 times extended time series of length 4M into the original M-point or 64-point transform of the original OFDM modulation and demodulation process.

EQUALIZATION: The envelopes of the shaped OFDM frames are 75% overlapped and successive frames may be correlated due to this time domain overlap. This correlation is denoted as intersymbol interference (ISI). Similarly, the spectral overlap of the adjacent channels in shaped OFDM result in correlation between adjacent channels and the correlation is denoted as adjacent channel interference (ICI). The overlap of the spectral main lobes is illustrated in figure 2 for the window and low-pass filter shaping envelopes.

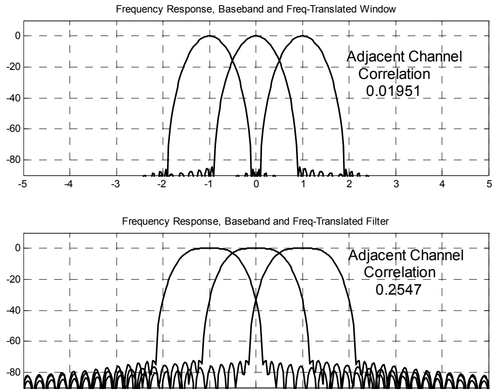

Figure 2. Spectra of Adjacent Shaped OFDM Channels: Base-Band Channel and First Sub-Carrier Channel

Due to this spectral overlap, a signal in a given spectral bin projects 0.01951 and 0.2547 into the adjacent spectral bin The overlap of adjacent shaped OFDM symbols is illustrated in figure 3 for the same shaped envelopes of figure 2.. The time series corresponding to these overlapped time intervals are not orthogonal signals as they were for the non-overlapped rectangle windowed time frames. Due to this overlap, a signal in a shaped OFDM frame projects 0.4957 and 0.00017 into the adjacent frames.

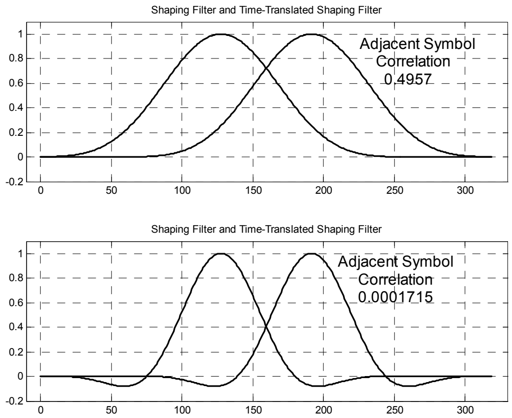

Figure 3. Time Series of Adjacent Shaped Overlapped Symbols. Adjacent Symbols Shifted N/4 or 64 Samples

Note that the window-shaped envelope has low ACI (0.02) and high ISI (0.50) while the filter-shaped envelope has high ACI (0.25) and low ISI (0.0001). By evolving from one type of shaping filter towards the other we can trade ACI for ICI and there is a system question about which mix is more desirable. The primary driver is the size, hence resource requirement, of the equalizer following the matched filter. The ACI due to spectral overlap and the ICI due to temporal overlap are treated as known channel distortion effects and we apply post process equalization at the output of the polyphase-matched filter or output of the FFT to suppress the interference and thus decouple the adjacent channels and adjacent time frames.

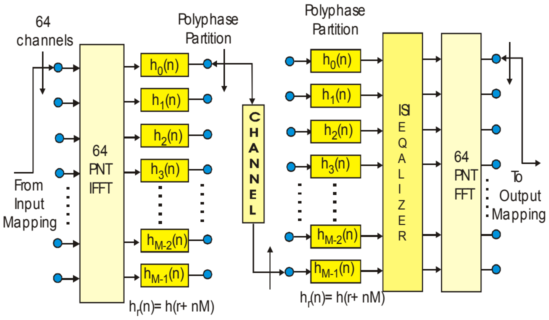

Figure 4. Primary Signal Processing Blocks in Shaped OFDM Modulator and Demodulator

The processing blocks required to implement the shaped OFDM modulator and demodulator is shown in figure 4. At the modulator we have the input IFFT to form the time series for each channel and the polyphase filter that shapes the envelope of the time frames while overlapping successive frames. The input and output of the processing blocks are subject to series-to-parallel (S-to-P) and parallel to series (P-to-S) formatting respectively. At the demodulator the received data is delivered to the matched filter through the S-to-P formatting as data vectors of length 64. Each successive 64-sample vector propagates through the matched filter to form the folded vector time series. This folded vector exhibits modulation generated ICI and ACI and is processed by the equalizer to suppress these known and undesired couplings. The equalized data, with an equivalent rectangle envelope is channelized by the FFT and then sent to the post processing blocks for detection and synchronization tasks.

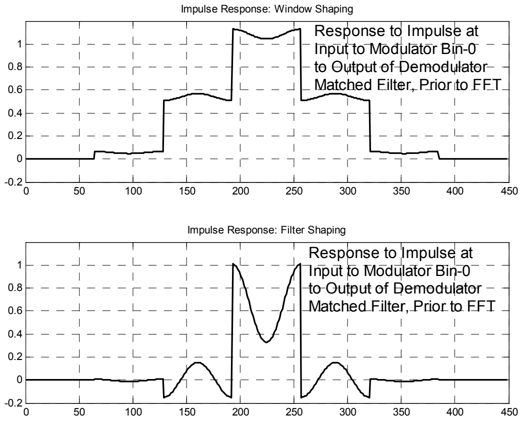

Figure 5. Impulse Responses from Modulator DC Bin to Output of Demodulator Matched Filter Output

Figure 5 presents the response of the processing chain from the DC input of the IFFT at the modulator to the output vectors from the matched filter at the demodulator. Each 64-point sequence is delivered to the FFT for final demodulation processing. Here the input to the modulator is an impulse at the DC bin. In the top subplot we see the ISI for window shaping as a sequence of 7-intervals of primarily DC blocks, which are demodulated as 7-samples in the DC bin of the receiver FFT. We note that each 'DC' block in the sequence also contains a single cycle of a low amplitude cosine. These cosine components project to the adjacent frequency bins of the FFT as the low level ACI of the process. In the lower subplot of figure 5 we see zero ISI for filter shaped OFDM as the 7-interval sequence contains a DC component in only the center interval. This sequence will form a single DC output bin in the subsequent FFT processing. All seven of the intervals are seen to contain significant levels of a single cycle of a cosine. These cosine components project to the adjacent frequency bins of the FFT as the ACI of the process.

We now have a clear picture of the equalizer chore required on each of the 64 paths of the demodulator polyphase filter. Each equalizer treats the three samples on either side of the center sample as pre and post echoes of a known multi path channel. These echoes are easily suppressed with deterministic zero forcing equalizers. Figure 6 presents the result of equalizing the impulse response sequence from the lower plot of figure 5. Here the equalizer has 19-taps per polyphase filter path. Interestingly, the shaping filter and the matched filter each have only 4-samples per polyphase path. Note the small residual notch in the center of each frame. The FFT of this equalized frame delivers the transmitted impulse to the DC bin along with low level ACI that is shown in the third subplot in figure 6. Here the ACI level is below 0.002 and this can be made arbitrarily small by the use of longer equalizers.

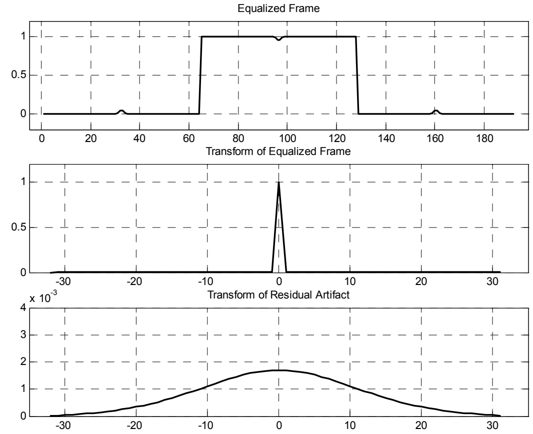

Figure 6. Frame Equalized with 19-tap Equalizer, Spectrum of Equalized Frame, and Spectrum of Residual Artifacts

Figure 7 illustrates the quality of the equalization afforded by a 19-tap equalizer in each polyphase filter path in the OFDM demodulator. Shorter equalizers allow the residual ISI and ACI to smear the constellation a greater amount than that shown in figure 7 with longer equalizers able to further reduce the residual ISI and ACI. Of course there is no need to suppress the residual artifacts below the thermal noise levels for which the modem and constellation density has been designed. The equalizer can reside at the input the output of the FFT, and the choice for its location is a driven by considerations of the ACI ICI levels.

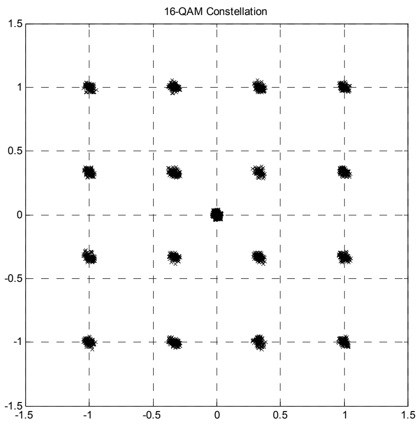

Figure 7. 16-QAM Constellation of Shaped OFDM Modulator and Demodulator

SYSTEM PERFORMANCE: The shaped OFDM signal was designed to contain a set of non orthogonal sub carriers with deep suppression of their spectral side lobe levels. The deep side lobe levels were introduced to keep a narrow band interference signal from projecting into the narrow band channels through their side lobes. By reciprocity, the shaped OFDM signals do not project into other spectral regions through their side lobe levels. Consequently, the shaped OFDM spectrum is tightly confined to the spectral span defined by the center frequencies of the narrowband channels. Figure 8 illustrates the spectrum obtained from the shaped OFDM modulator. The adjacent channel levels are seen to be down 80 dB one narrow bandwidth channel away from the allocated signal bandwidth.

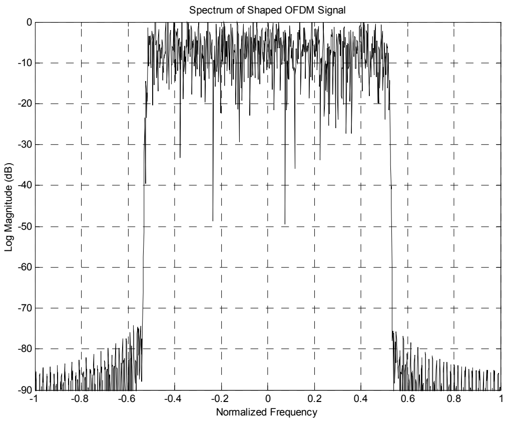

Figure 8. Spectrum of Shaped OFDM Modulated Signal

The shaped OFDM signal not only supports tight packing of the narrowband channels within its bandwidth, it also enables tight packing of adjacent channels. For comparison, figure 9 presents, on the same scale used for figure 8, the spectrum obtained from a standard OFDM modulator. This spectrum exhibits the high-level side lobes of the basis signals of a DFT, the default or rectangular windowed sinusoids with an integer number of cycles per window interval.

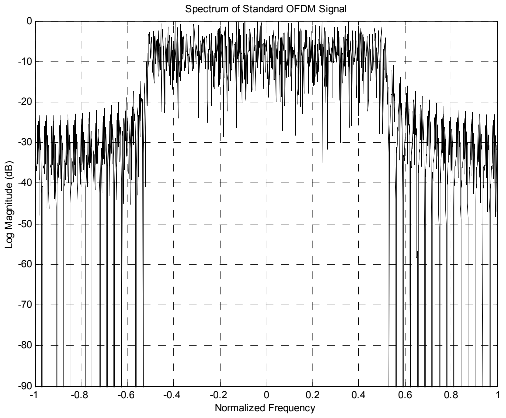

Figure 9. Spectrum of OFDM Modulated Signal

While on the topic of jammer tolerance, figure 10 shows the demodulated power spectrum of a shaped OFDM signal with an in-band jammer 23 dB above the modulation signal level. The tone frequency is at the mid frequency, or band edge, between the centers of two narrow band OFDM channels

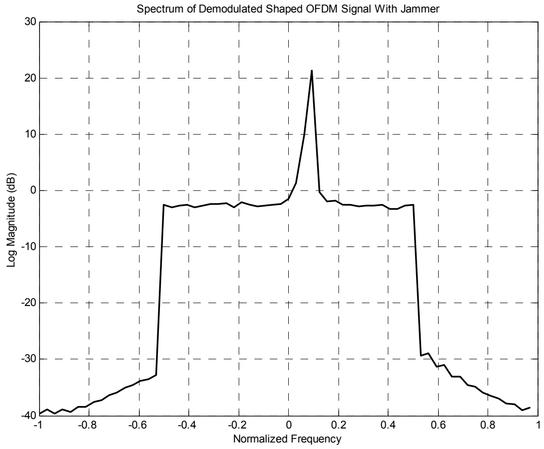

Figure 10 Power Spectrum of a Shaped OFDM with 23 dB In Band Jammer

We note that the constellation structure is preserved in spite of the presence of the jammer. To better see the effect of the jammer on the shaped OFDM signal, we formed a set of shaped OFDM frames with only two carriers per frame.

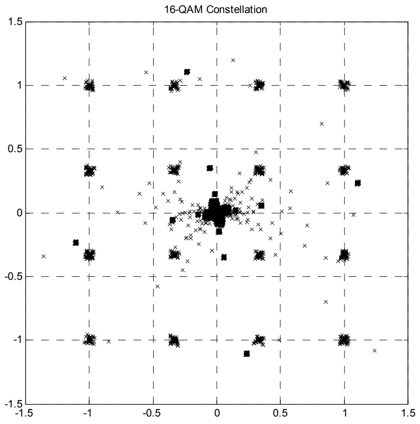

Figure 11 Constellation for 16-QAM Shaped OFDM with 23 dB In Band Jammer

Figure 12 shows the spectrum of the sparse signal set and we can easily see how the side lobe structure of the non-bin centered interference is distributed through the signal bandwidth. Be sure to compare the side lobe structure of this signal to the side lobe structure of the standard OFDM signal shown in figure 9.

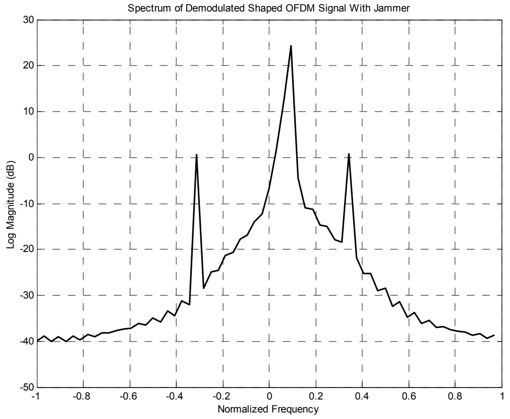

Figure 12 Power Spectrum of a Sparse Shaped OFDM with 23 dB In Band Jammer

Figure 13 shows the constellation obtained from the two occupied channels. The dense constellation mass at the origin is the collection of zero-values narrow band signals. Here we see that most of the constellation points are related to the two occupied channels and the large number of unoccupied channels. The scattered constellation points are due to the channels whose edges are adjacent to the spectrum of the interfering tone

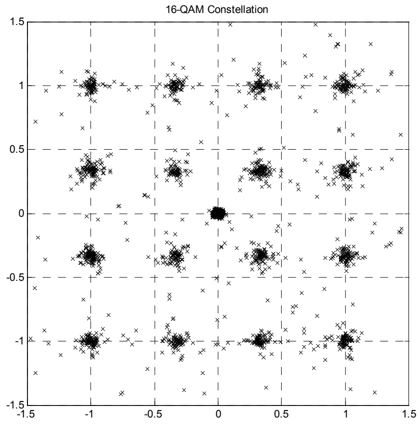

Figure 13 Constellation for Sparse 16-QAM Shaped OFDM with 23 dB In Band Jammer

CONCLUSIONS: We have presented the structure of a shaped OFDM modulation scheme that exhibits deeply attenuated spectral side lobes. This property confines the signal spectra to a tight spectral mask while simultaneously decouples the narrowband channels from in band and near band narrow band jammers. The shaping that accomplishes this task requires increased frame length to preserve the spectral occupancy and overlap of the frames to maintain data throughput. An equalizer suppresses ACI and ISI resulting from the time and spectral overlap of the narrowband signal set.

REFERENCES:

fred harris and Chris Dick, "A Modified Polyphase Transform with Embedded Fractional Sample Rate Change" International Signal Processing Conference, Dallas, Texas, March 31 - April 3, 2003.

fred harris, Chris Dick, and Michael rice, "Digital Receivers and Transmitters Using Polyphase Filter Banks for Wireless Communications", Special Issue of Microwave Theory and Techniques, S4-8 MTT, March 2003

fred harris and Hana Abusalem, "Comparing Traditional FFT Based Frequency Domain Excision with Polyphase Transform Excision" Institute of Navigation Conference (ION-99), 28-June, 1-July 1999, Boston, MA

fred.harris and Wade Lowdermilk, "Design and performance of fading insensitive Orthogonal Frequency Division Multiplexing (OFDM) using Polyphase Filtering Techniques", 30th Asilomar, Conference on Signals, Systems, and Computers, Pacific Grove, CA., 28–30 Nov. 1996.