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ADAPTIVE FILTERS USING FRACTAL DIMENSION OF DATA

ABSTRACT

The fractal dimension of a data series can be used as a sensitive indicator of the temporal variability of a signal. Smooth continuous signals have fractal dimension of one while signals with discontinuities and very noisy signals have fractal dimension between one and two. This measure, estimated over short sliding intervals, can be used to adjust the local spectral characteristics of a digital filter to emphasize or to suppress local temporal features of the signal. We have examined and have compared the performance of a number of filters with parameters controlled by the short term fractal dimension of two classes of signals. The processed signals were sinusoids and square waves with varying amounts of additive white Gaussian noise.
The filter structures included the finite impulse response (FIR), infinite impulse response (IIR), and edian filters. We examined two methods of changing the filters with fractal dimension; these were, changing between fixed filters as the fractal dimension crossed thresholds, and changing filter parameters proportionally with the fractal dimension. In addition to describing the filtering process we also report on methods for estimating the fractal dimension over the sliding Interval.

INTRODUCTION

An important application of filtering is the separation of desired signals from undesired signals. The undesired signal can be noise or interference. Separation may be possible if the signals exhibit distinguishable characteristics such as different center frequency, different bandwidths, or unique temporal phase structures. For instance, the effect of broadband noise on a spread spectrum digital communication system is minimized by matching the filter to the known temporal phase structure.

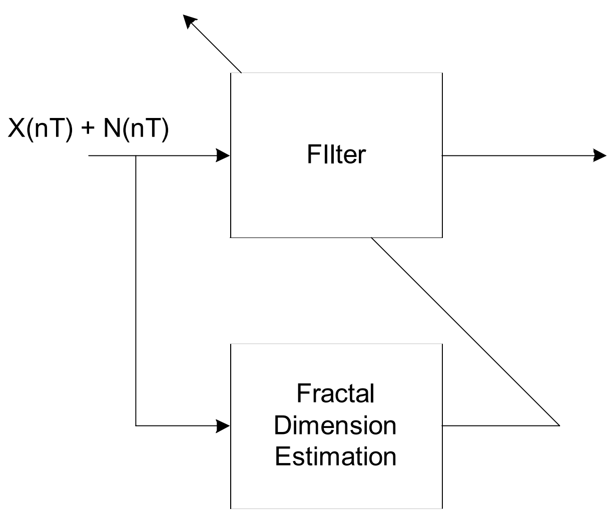
When the signal is a random variable the standard procedure is to use the filter to limit the noise bandwidth to the signal's average bandwidth. This is done, for instance, in commercial AH radio which deliver speech and music over a nominal 5-KHz bandwidth. The problem with this approach, is that the signal may not be stationary and may occupy different bandwidths over different time intervals.

Similarly, for many channels, the undesired signal may be adjacent channel interference which is another non stationary signal. Speech is a prime example of such a non stationary signal. The mute switch on a communication receiver is mute evidence of the designer's awareness of this fact.

Various data adaptive filtering techniques have been developed to take advantage of the non stationary structure of signals, noise, and interference. Well known examples are Dolby Noise Reduction which compands via a signal level dependent frequency boost followed by a complementary reduction and Dynamic Noise Reduction (DNR) which uses a variable bandwidth lowpass filter to reject the time varying the noise-only bandwidth of non stationary signal sources.

Data dependent filters which respond to non stationary temporal structure (as opposed to spectral content) such as random impulses are the pop-filter used for vinyl records and the median filter used to reduce salt-and-pepper noise in images.

In this paper we present a class of filters which monitor a different measure of temporal non stationarity and alters their bandwidth in response to this measure. The measure we have examined is the fractal dimension of the data. A simple model of this filter structure is shown in Figure 1.



■ Figure 1. *Reduced-complexity FIR filtering*

FRACTALS AND FRACTAL DIMENSION

Mandelbrot [1] developed fractal geometry to portray many of the irregular and fragmented patterns which occur in nature. Examples of patterns which exhibit fractal-like properties are the fern leaf, turbulent gas flow, clouds, and non stationary noise on communication channels. Fractal dimension, a measure used to describe fractal patterns, can ba applied to both fractal and non-fractal shapes. A fractal is a pattern with non-integer fractal dimension and a non-fractal pattern is one with integer fractal dimension.

A pattern's fractal dimension describes how completely the pattern fills the space in which it resides. Consider for instance, two squares, one completely filled-in and the other with sparsely occurring points. The sparsely occupied square has lower fractal dimension because it occupies less of its space than the filled-in square.

Fractal shapes are said to be self-similar because they tend to have the same roughness and sparseness regardless of the level of magnification used to view them. Self-similarity means that the fractal dimension describes the sparseness at all levels of magnification levels. To determine the fractal dimension of a pattern we first cover the pattern with a number N(p) of small objects of various sizes p. The fractal dimension P is related to these parameters as indicated in (1a). The Dimension is than computed as show in (1b).

$$\frac{N(p_2)}{N(p_1)} = \left|\frac{p_1}{p_2}\right|^D \tag{1a}$$

$$D = \frac{\log\left[N(p_2)/N(p_1)\right]}{\log(p_1/p_2)} \tag{1b}$$

Using a line segment of 10.0 meters as an example and setting p1 and p2 to 1.0 and 0.1 meters respectively, we determine that N(p1 is 10 and N(p2) is 100. When these parameters are substituted in (lb) we obtain (1c) which, as expected, indicates that the line has dimension 1.0.

$$D = \frac{\log\left[100/10\right]}{\log\left[1.0/0.1\right]} = 1.0 \tag{1c}$$

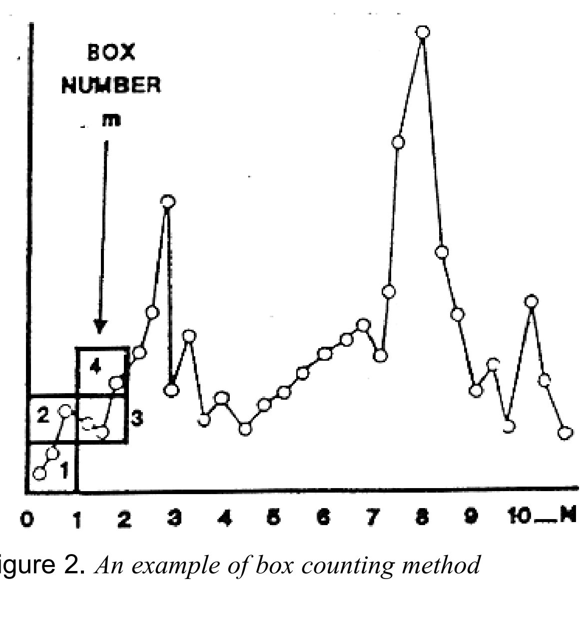
Using a square of 10.0 meters on a side, and setting p1 and p2 to be covering aquares of 1.0 and 0.1 meters per side, we determinate that N(p1) and N (p1) are 100 and 10.000 respectively.

Substituting these parameters on (1b) we obtain (1d) which, again as expected, shows that the square has dimension 2.0.

$$D = \frac{\log[10{,}000/100]}{\log[1.0/0.1]} = 2.0 \tag{1d}$$

Natural fractals, such as a shore line, lack the true regularity of an algorithmic structure but nevertheless are self similar in a statistical sense. Thus in order to determine the fractal dimension of natural shapes we must average the measured fractal dimension made over different scales.

The method of calculating the fractal dimension for natural shapes starts with the box counting method. We simply count the number of just touching boxes of dimension p required to cover the shape. This idea is shown in Figure 2. for the indicated size box. The fractal dimension is determines as the average slope of log[H(p)] versus log[p] [2] for various box sizes p.



■ Figure 2. *An example of box counting method*

Sampled data systems usually consist of a set of points exhibiting equal spaced increments in time (or space]. To calculate the fractal dimension these points are connected with straight lines to form a continuous curve and the box counting method can then be applied. A slight variation of the box counting method can be used which does not require us to play "connect the dots". This method takes advantage of the fact that the data samples are equally spaced. In this method intervals of width "p" are defined on the independent (time) axis. The data string y(t) in each interval is searched for the maximum and minimum values. An estimate of the number of boxes n1, (p) required to cover the curve in the i-th interval is shown in (3).

$$n_1(p) = \frac{y_{max} - y_{min}}{p} \tag{3}$$

The total number of boxes required to cover the curve is the sum of the box counts taken over all intervals. The fractal dimension is then determined as the slope of log [n1 (p)] versus log [p]. Thus to compute the fractal dimension over two bow widths the number of boxes n1 (p) from t1 to t1+p and from t1+p to t1+2p is first determined. Then the number of boxes from t1 to t1+2p is determined. Calling the first two counts n1 and n1 and the last count n3 , the fractal dimension is show in (4)

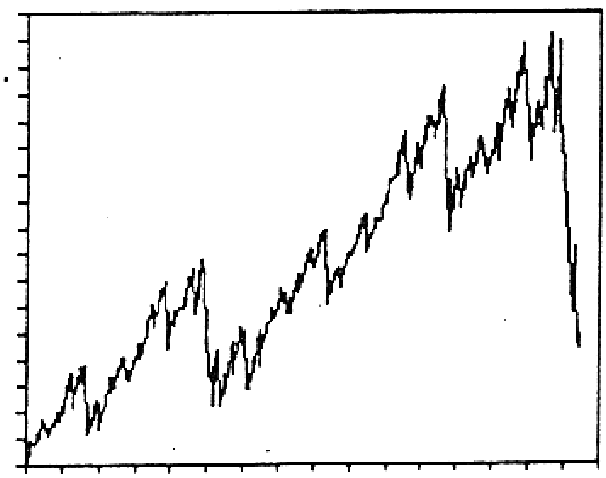
$$D = \frac{\log[n_3] - \log[n_1 + n_2]}{\log[2]} \tag{4}$$

To test this method the fractal dimension was estimated for fours data sets generated with theoretical fractal dimensions via an iterated function systems. The results of this experiment are shown in Table. 1. As seen, the estimate obtained by this technique is biased, being slightly lower than the theoretical dimension

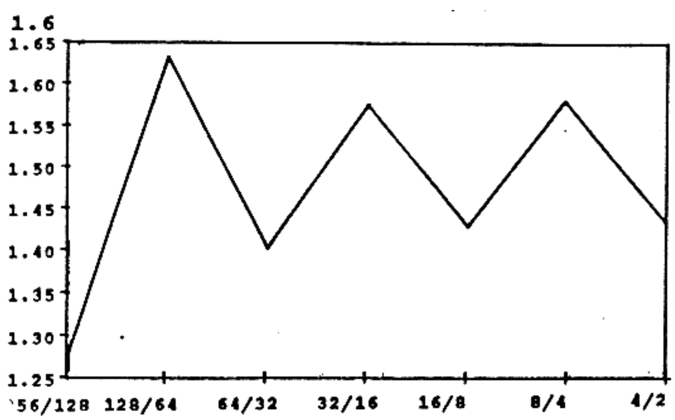
THEORETICAL DIMENSION	CALCULATED DIMENSION
1.1	1.085
1.3	1.253
1.6	1.536
1.9	1.828

■ Table 1. *Theoretical and calculated dimension*

Figure 3a presents an example of a data set formed with dimension 1.6 while figure 3b shows the estimated fractal dimension formed for the indicated pair of box widths.



■ Figure 3a. *Data series of fractal dimension*



■ Figure 3b. *Estimated fractal dimension obtained with modified box counting method*

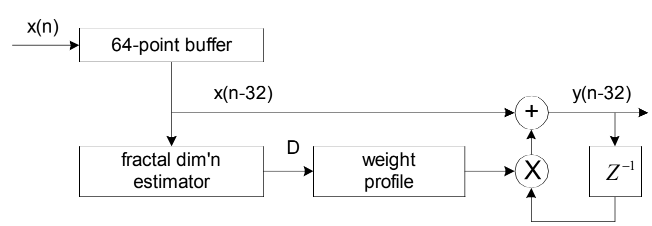
ADAPTIVE FILTERING USING FRACTAL DIMENSION

The design specifications of frequency selective filters requires a priori information about the statistics of the signal, noisE, and interference of the data to be processed. The filter is optimal when the statistical characteristics of the signal match the a priori information on which the filter design is based. When a priori information about the statistics is not avail- able, or when those statistics change with time, we call upon data adaptive filters. These filters are self designing in that when operating they estimate the appropriate statistics from the data and adjust internal, parameters in response to those estimates. This structure was suggested in Figure 1.

There are numerous control strategies we can invoke to change the parameters of a data adaptive filter. In addition, there are many possible structures of the underlying filter whose coefficients are to be changed. We also have a large class of signal and noise structures we can use to probe the performance of the adaptive filters. To keep the size of this task manageable we have selected a small set of what we consider reasonable signals and filter structures, as well as reasonable but somewhat arbitrary control law strategies.

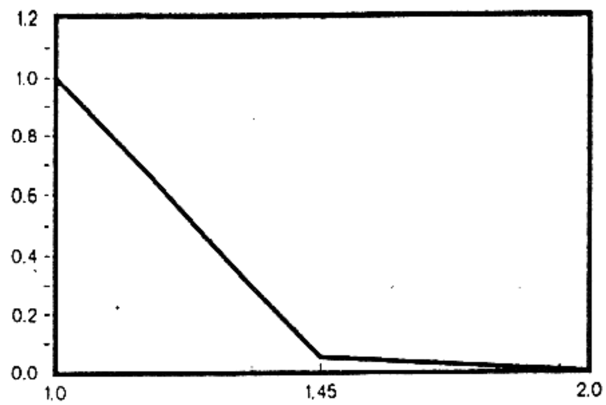
Common to each filter is an auxiliary short time sliding processing window used to form a running estimate of the input data's fractal dimension. The selected window was unweighted (ie. uniform) and of length 64 points placed symmetrically about the signal time sample of interest. The box sizes used for the box counting method were of length 16 and 8.

Three types of filters were investigated. The first is an adaptive IIR filter whose single weight is varied continuously with the data's fractal dimension in accord with a specified profile. This filter and control law profile is shown in Figure 4.



■ Figure 4a. *Adaptive IIR filter*

The second filter is an adaptive lowpass FIR filter with a COS-SQUARED impulse response of length switchable between 11 and 6l points. in this law, the short filter is selected when the fractal dimension ia below a (reasonable) threshold. The decision threshold was set at 1.2 for sinusoidal inputs and at 1.5 for rectangular pulse train inputs.



■ Figure 4b. *Control law for IIR filter weight*

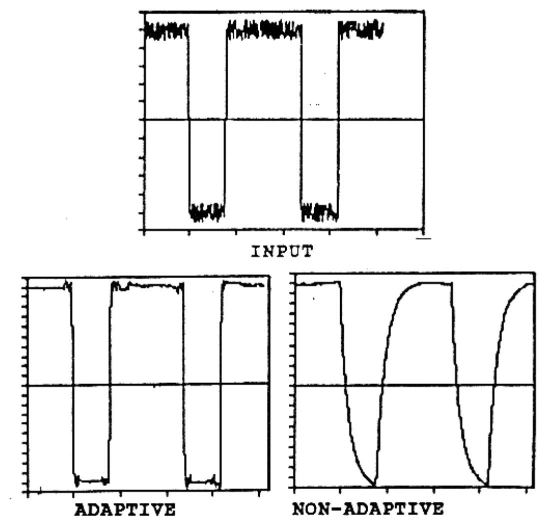
The third filter examined was an adaptive non-linear median filter of length switch-abla between 15 and 41 points. Here, as in the adaptive FIR filter, the short length ia selected when the fractal dimension is below the threshold. The decision threshold levels were the same as those used for the FIR filter.

FILTERING RESULTS

The response of the adaptive filters described in the previous section were examined and compared against non adaptive versions using two classes of input signals. The classes were sine waves and square waves width varying amounts of additive white noise with uniform and Gaussian distributions. These signals were selected to test the filters width signals exhibiting nearly constant fractal dimension (sine wave) and with signals exhibiting wide variation of fractal dimension (square wave).

Figure 5. shows the input and output signals for the adaptive and the non-adaptive IIR filters. Here the signal is a square wave width uniformly distributed noise having a peak noise amplitude equal 10% of the square wave amplitude. Al clearly shown, the adaptive IIR essentially eliminated the noise without effecting the sharp transitions of the square wave. The non-adaptive IIR also eliminated the noise but was not able to preserve the transitions of the square wave. These curves closely match results reported by Strahle [3].

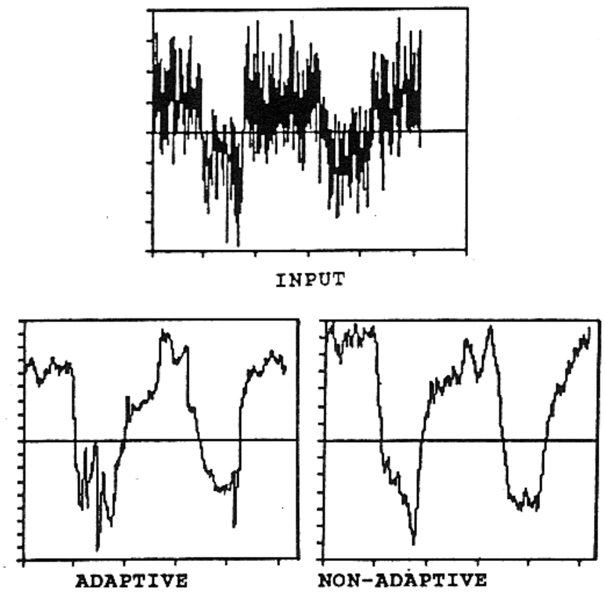
Figure 6. shows the input and out put for the same adaptive and the non-adaptive IIR filters. Here the signal is a square wave with -l.1 dB SNR of additive white Gaussian noise. We use this as the



■ Figure 5. *Input and output for adaptive and non adaptive IIR filters, (high SNR)*

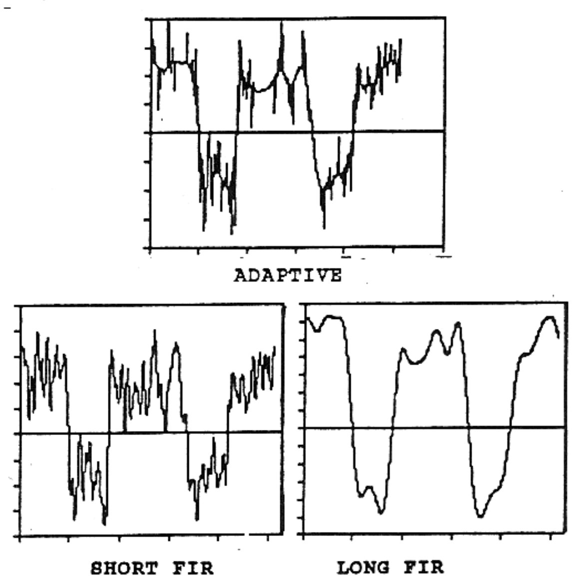
common test signal so we can compare the performance of all the filters. The fractal dimension of the noisy data is such that the filter is adjusting its coefficient between 0.0 and 0.05. In general, for large and medium SNRs the adaptive filter outperforms the non-adaptive filter and for low SNRs the filter performances are identical.

Figure 7. shows the outputs for the adaptive and for both short and long non - adaptive FIR filters. The input signal is the same used to test the IIR filter (see figure 6.) .The fractal dimension of low level noise is less then the selected switching threshold except in the neighborhood of the tran-



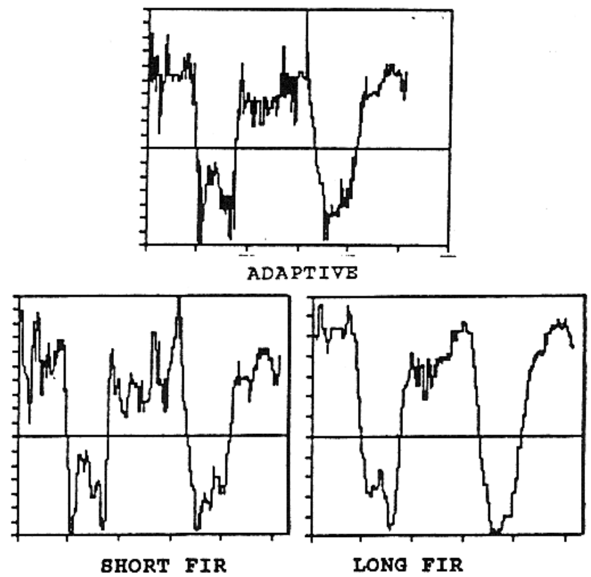
■ Figure 6. *Input and output for adaptive and non adaptive IIR filters, (-1.1 Db SNR)*

sitions. For high level noise the fractal dimension always exceeds the threshold. Thus for large SNRs the short and the adaptive FIR outputs are identical while for low SNRs the long and adaptive FIR outputs are the same. For intermediate SNRs the Adaptive filter outperforms both the short and the long FIR.



■ Figure 7. *Outputs for adaptive and non adaptive IIR filters, (-1.1 Db SNR)*

Figure 8. shows the outputs for the adaptive and for both short and long non-adaptive median filters. The signal is the same square wave defined earlier for the FIR filters (see Figure 6). Since the threshold conditions are the same as for the FIR filter, comments made about the FIR filter generally apply here also.



■ Figure 8. *Outputs for adaptive and non adaptive median filters, (-1.1 Db SNR)*

CONCLUSIONS

The technique of using the fractal dimension of input data to adapt a filter's characteristics is a viable one. This class of adaptive filters generally improved the output SNR for input signals which contain desired high frequency characteristics and undesired broadband noise. The square wave in noise is the prime example of this type of signal. We alluded performance tests of these filters against sine waves in noise but due to limited space have not demonstrated results. To be complete we report that the filters, as described do not appear useful for improving the ability to discriminate sinusoids in noise.

We have presented three structures, selected somewhat arbitrarily, to demonstrate forms data dependent filters might take. We think that other variants of the fractal dimension dependent filter structure should be examined. These variants might include optimum parameter variation profiles with discriminant, continuously adjustable FIR structures as opposed to switched forms, and combinations of adaptive and non-adaptive structures.

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